Kaon semileptonic decay form factors from lattice QCD

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$|V_{us}|$ from K_{l3} decays

$$|V_{us}| = \sqrt{\frac{128\pi^3\Gamma(K_{l3})}{G_F^2 M_K^5 S_{EW} I_K(\lambda_+)}} \cdot \frac{1}{f_+^{K\pi}(0)}$$

- high statistics experiments (E865, KLOE, KTeV, NA48) provide *Br*(K_{l3}) with ≤1% accuracy
- dominant uncertainty comes from the theoretical estimate of the vector form factor $f_+(0)$

$$\langle \pi(p')|V_\mu|K(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- reliable calculation from the first principle of QCD is highly desirable

$f_+(0)$ from chiral perturbation theory

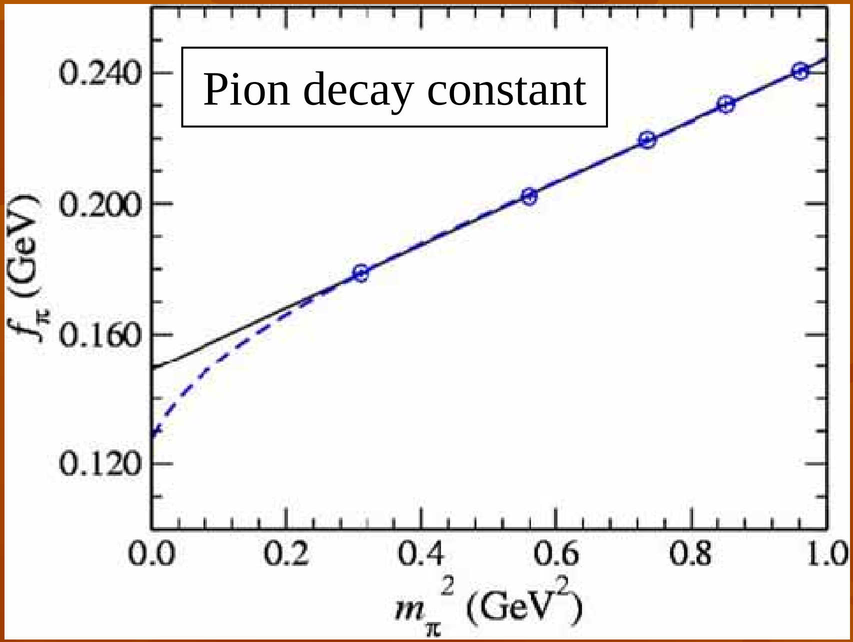
- $f_+(0)=1$ in the SU(3) limit
  - estimation of corrections from isospin breaking, SU(3) breaking is the key
- Leutwyler and Roos (1984)
  - $f_+(0)=1+f_2+f_4=1-0.023-0.016(8)=0.961(8)$
  - f_2 is unambiguously determined by f_π and m_{PS} from ChPT
  - rough estimate of f_4 using a model of the wave function
- higher order and radiative correction
  - ChPT at $O(p^6)$ and electromagnetic correction at $O(e^2p^2)$
  - need better experimental resolution to determine LECs

$f_+(0)$ from lattice QCD

- Becirevic et al.
  - Nf=0, O(a)-improved Wilson + plaquette
  - $f_+(0)=1+f_2+f_4^q$=1-0.023-0.017(5)(7)=0.960(9)
- RBC collaboration
  - Nf=2, domain-wall quark + DBW2
  - $f_+(0)$=0.955(12)
- Fermilab/MILC/HPQCD collaboration
  - Nf=2+1, improved staggerd + Symanzik
  - $f_+(0)$=0.962(6)(9)

JLQCD Nf=2 configurations

- gauge configurations include internal loop effects of up and down quarks
- Non-perturbatively O(a)-improved Wilson fermion + plaquette gauge
- 20^3x48, β=5.2, a^{-1}~0.1 fm
- 5 quark masses correspond to m_π=550~1000 MeV
- 1,200 configs separated by 10 HMC trajectories
- light hadron spectrum, decay constant and quark masses / *B* meson decay constant and Bag parameters



3-step strategy to calculate $f_+(0)$

$$f_+(0) = f_+(q^2_{\max})\left[1+\xi(q^2_{\max})\frac{m_K-m_\pi}{m_K+m_\pi}\right]\times\frac{f_+(0)\left[1+\xi(0)\frac{m_K-m_\pi}{m_K+m_\pi}\right]}{f_+(q^2_{\max})\left[1+\xi(q^2_{\max})\frac{m_K-m_\pi}{m_K+m_\pi}\right]}\times\frac{1}{\left[1+\xi(0)\frac{m_K-E_\pi}{m_K+E_\pi}\right]}$$

1. determine f_0 at $q_{max}{}^2=(m_K-m_\pi)^2$
2. interpolate to $q^2=0$
3. subtract unnecessary contribution from $\xi(0)=f_-(0)/f_+(0)$

To achieve a few percent accuracy,
a set of double ratio of correlation functions is used
→ renormalization factors and bulk of statistical fluctuations cancel

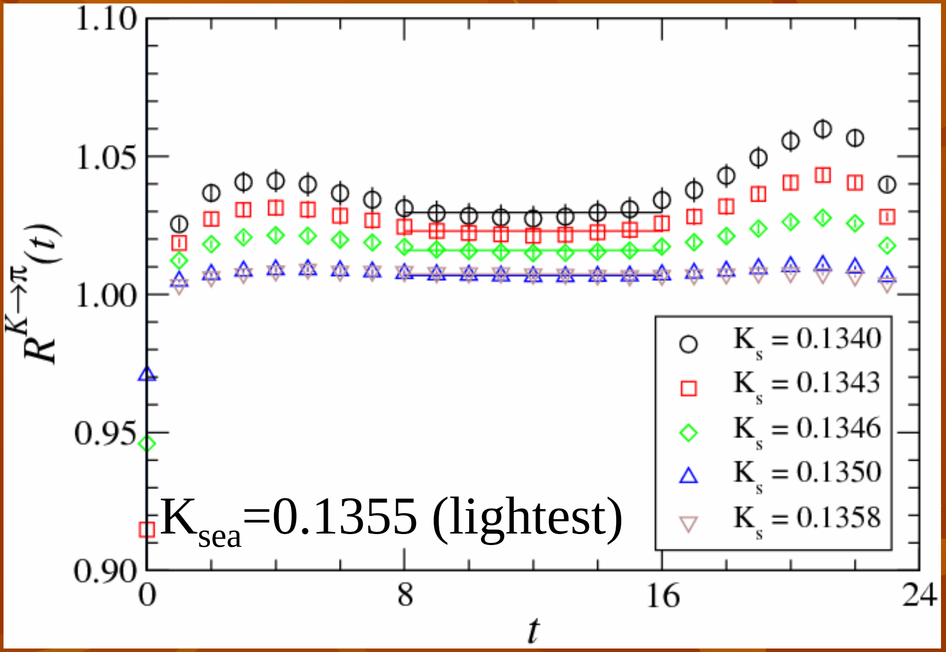
Step 1: determine $f_0(q_{max}^2)$

$$R_1(t) = \frac{C_{\pi V_4 K}(t,T/2;\vec{0},\vec{0})\,C_{KV_4\pi}(t,T/2;\vec{0},\vec{0})}{C_{\pi V_4 \pi}(t,T/2;\vec{0},\vec{0})\,C_{KV_4 K}(t,T/2;\vec{0},\vec{0})} \to \frac{\langle \pi(0)|V_4|K(0)\rangle\langle K(0)|V_4|\pi(0)\rangle}{\langle \pi(0)|V_4|\pi(0)\rangle\langle K(0)|V_4|K(0)\rangle} = \left[\frac{m_K+m_\pi}{2\sqrt{m_K m_\pi}} f_0(q_{\max}^2)\right]^2$$

three-point function

$$C_{KV_\mu\pi}(t_x,t_y;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}} \langle \pi(t_y,\vec{y})V_\mu(t_x,\vec{x})K(0)\rangle e^{+i\vec{q}\cdot\vec{x}}e^{-\vec{p}\cdot\vec{y}}$$

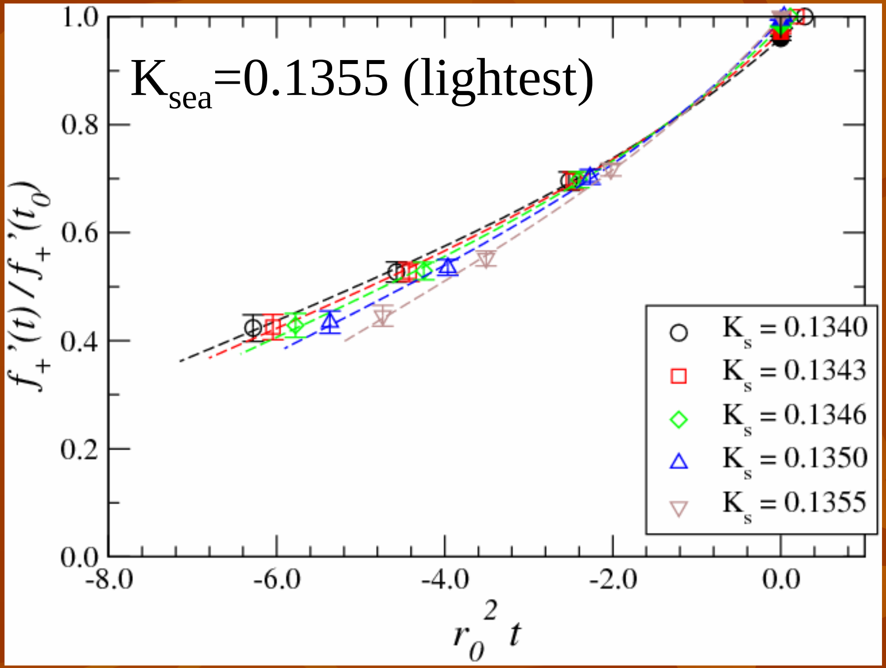
- Double ratio used before for semileptonic *B* decay by the Fermilab group
- measures SU(3) breaking at $q_{max}^2=(m_K-m_\pi)^2$
- larger deviation from 1 for larger mass differences



Step 2: interpolate to q^2=0

$$R_2(t;\vec{p}) = \frac{\dfrac{C_{\pi V_4 K}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 K}(t,T/2;\vec{0},\vec{0})}}{\dfrac{C_{\pi\pi}(t;\vec{p})}{C_{\pi\pi}(t;\vec{0})}} \to \frac{\dfrac{\langle \pi(p)|V_4|K(0)\rangle}{\langle \pi(0)|V_4|K(0)\rangle}}{\dfrac{\langle \pi(p)|P|0\rangle}{\langle \pi(0)|P|0\rangle}} = \frac{m_K+E_\pi}{m_K+m_\pi}\,\frac{f_+(q^2)\left[1+\xi(q^2)\dfrac{m_K-E_\pi}{m_K+E_\pi}\right]}{f_+(q^2_{\max})\left[1+\xi(q^2_{\max})\dfrac{m_K-m_\pi}{m_K+m_\pi}\right]}$$

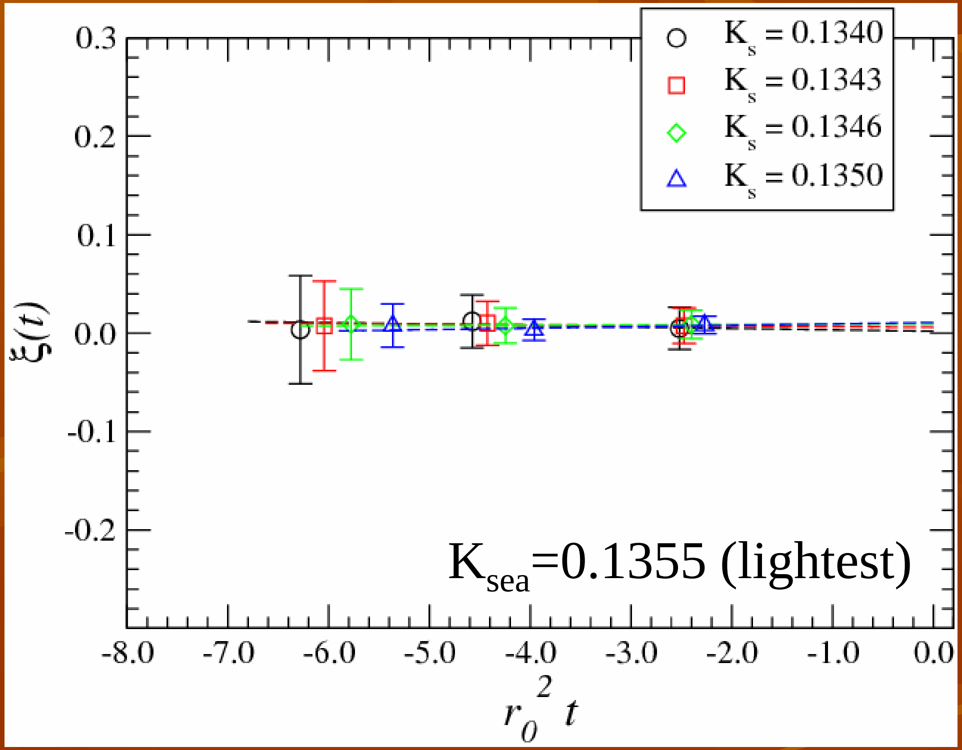
- 2pt. function in the denominator is to cancel the energy mismatch
- exactly 1 in zero-recoil case
- interpolate to q^2=0 with a quadratic function



$$t = q^2 = (m_K - E_\pi)^2 - |\vec{p}|^2, \quad \left(|\vec{p}| = \frac{2\pi}{20}, \frac{2\pi}{20}\sqrt{2}, \frac{2\pi}{20}\sqrt{3}\right)$$

Step 3: subtract ξ contribution

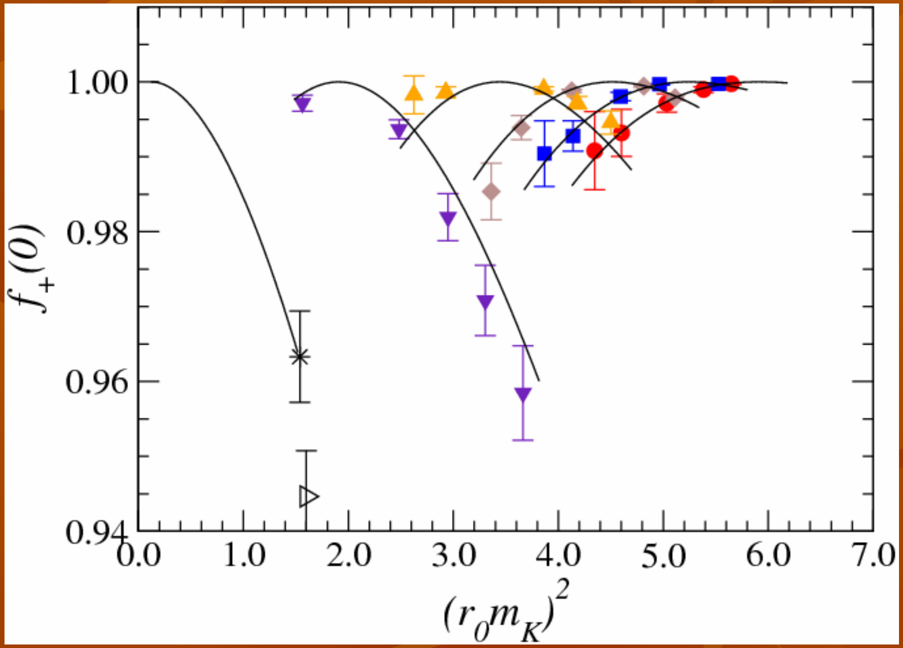
$$R_3(t;\vec{p}) = \frac{\dfrac{C_{\pi V_i K}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 K}(t,T/2;\vec{p},\vec{p})}}{\dfrac{C_{\pi V_i \pi}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 \pi}(t,T/2;\vec{p},\vec{p})}} \to \frac{\dfrac{\langle \pi(p)|V_k|K(0)\rangle}{\langle \pi(p)|V_4|K(0)\rangle}}{\dfrac{\langle \pi(p)|V_k|\pi(0)\rangle}{\langle \pi(p)|V_4|\pi(0)\rangle}} = \frac{1-\xi(q^2)}{\dfrac{m_K+E_K}{m_\pi+E_\pi}+\xi(q^2)\dfrac{m_K-E_K}{m_\pi+E_\pi}}$$



- q^2 dependence is very small and seems to be independent of the strange quark mass
- extrapolation to $q^2=0$ is done by assuming linear dependence

Chiral extrapolation

- the chiral logarithm is significant only in the region where $m_\pi^2 << m_K^2$, while the data region $½ < m_\pi^2/m_K^2 < 2$ is well described by the quadratic form
- partially quenched ChPT formula by Becirevic et al. is used



$$f_2^{pq} = -\frac{2m_K^2 + m_\pi^2}{32\pi^2 f^2} - \frac{3m_K^2 m_\pi^2 \ln\frac{m_\pi^2}{m_K^2}}{64\pi^2 f^2 (m_K^2 - m_\pi^2)} + \frac{m_K^2 (4m_K^2 - m_\pi^2) \ln\left(2 - \frac{m_\pi^2}{m_K^2}\right)}{64\pi^2 f^2 (m_K^2 - m_\pi^2)}$$

$$f_+(0) = f_+^{pq}(0) - f_2^{pq} + f_2 = 0.945(6) \text{ (preliminary)}$$

Results for $f_+(0)$

Leutwyler and Roos (1984)	0.961(8)
Becirevic et al. (2005)	0.960(9)
This work (quad.)	0.967(6)
This work (ChPT + quad.)	0.952(6)
This work (pqChPT + quad.)	0.945(6)

Vector charge radius

- charge radius is a slope of the form factor near *$q^2=0$*

$$f_+^{K\pi}(q^2) = 1 + \frac{1}{6}\langle r^2 \rangle_V^{K\pi} q^2 + \cdots$$

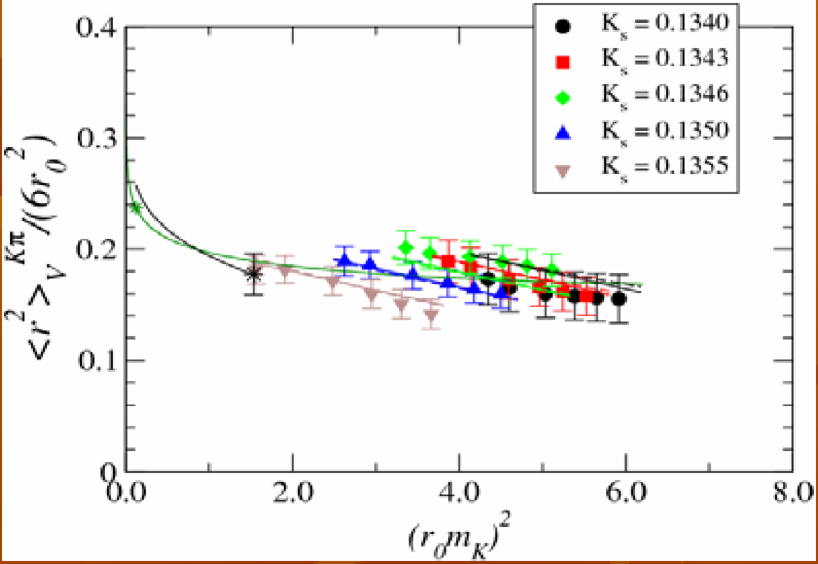
- one-loop ChPT predicts

$$\langle r^2 \rangle_V^{K\pi} = \langle r^2 \rangle_V^{\pi} - \frac{1}{64\pi^2 f^2}\left[3h_1\left(\frac{m_\pi^2}{m_K^2}\right) + 3h_1\left(\frac{m_\eta^2}{m_K^2}\right) + \frac{5}{2}\ln\frac{m_K^2}{m_\pi^2} + \frac{5}{2}\ln\frac{m_\eta^2}{m_K^2} - 6\right]$$

$$\langle r^2 \rangle_V^{\pi} = \frac{12L_9}{f^2} - \frac{1}{32\pi^2 f^2}\left[2\ln\frac{m_\pi^2}{\mu^2} + \ln\frac{m_K^2}{\mu^2} + 3\right]$$

- we fit the data with ChPT + linear

$$\langle r^2 \rangle_V^{K\pi} = 0.26(3)\ \mathrm{fm}^2\ \text{(preliminary)}$$



cf) the exp. value

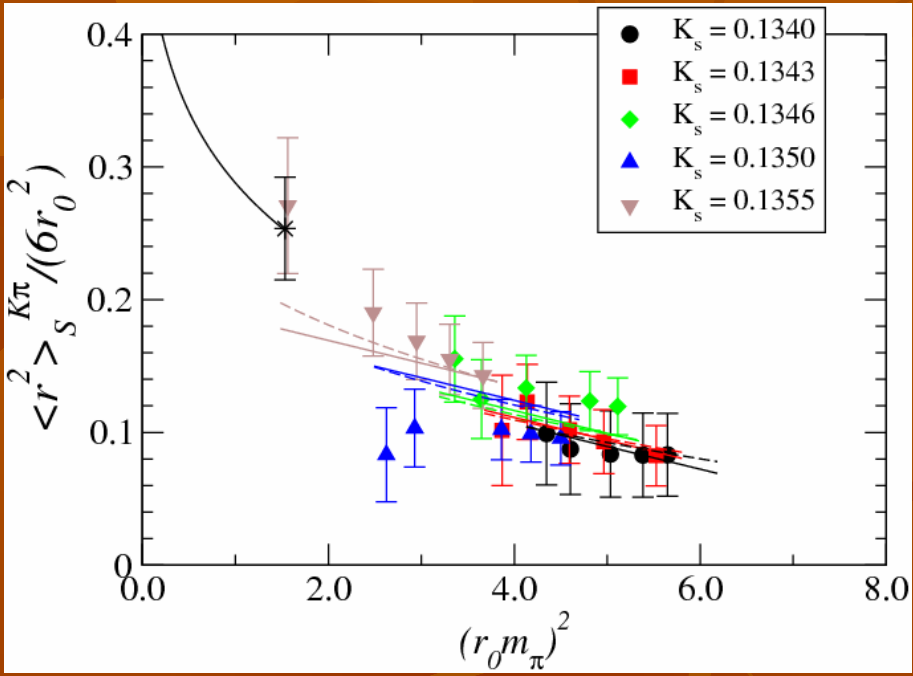
$$\langle r^2 \rangle_V^{K\pi} = 0.331(8)\ \mathrm{fm}^2$$

Scalar charge radius

- The same analysis can be applied to the scalar charge radius

$$\left\langle r^2 \right\rangle_S^{K\pi} = 0.37(6)\ \mathrm{fm}^2\ \text{(preliminary)}$$

- overshoots the exp. value 0.21(3) fm (PDG2004)



Summary

- lattice QCD enables us to calculate the kaon form factor from the first principle of QCD
- a few percent statistical accuracy can be achieved by the double ratio method
- study of systematic errors should be made, especially, much lighter sea quarks are necessary for reliable chiral extrapolation
- details are presented in hep-lat/0510068